Ends Pressure Losses in Extrusion of Polymer Melts Through Dies

INTRODUCTION

Early investigators of extrusion of polymer melts generally considered all of the pressure losses to occur within the die.[1,2] However, the researches of Mooney[3,4] and notably Bagley[5] made clear that the pressure loss p_T for extrusion through the die could be expressed as the sum of a term proportional to the length L of the die and a second term independent of die length. These may be termed die and ends pressure losses and may be expressed (for a cylindrical die) as

$$p_T = \Delta p_{\text{die}} + \Delta p_{\text{ends}} \tag{1a}$$

$$= 4(\sigma_{12})_w \frac{L}{D} + \Delta p_{\text{ends}} \tag{1b}$$

here D is the die diameter and $(\sigma_{12})_w$ is the shear stress on the die wall. By the late 1950s, there was general acceptance of the existence of Δp_{ends} in polymer melts, and eq. (1b) was used to correct capillary viscometer data. Generally, Δp_{ends} was considered to represent a pressure loss at the die entrance.

In the 1960s, Sakiadis[6] with polymer solutions and later Mori and Funatsu,[7] Arai,[8] and Han, Charles, and Philippoff[9] for polymer melts noted that if pressure was measured at various positions along the length of a die, the values did not extrapolate to atmospheric pressure but to a positive value Δp_{exit}. It thus became clear that Δp_{ends} was the sum of both entrance and exit pressure losses

$$\Delta p_{\text{ends}} = \Delta p_{\text{ent}} + \Delta p_{\text{exit}} \tag{2}$$

In this note, we will be addressing ourselves to the relative and absolute values of Δp_{ent} and Δp_{exit}.

The first attempt to interpret the value of Δp_{ent} was by Philippoff and Gaskins[10] whose ideas were based on a mechanical energy balance. They related $\Delta p_{\text{ent}} Q$, where Q is the extrusion rate, to the sum of the recoverable elastic energy during flow and entrance region viscous dissipation. They conclude that

$$\Delta p_{\text{ent}} = (2n + s_w)(\sigma_{12})_w \tag{3a}$$

where n is a small correction of type observed for Newtonian fluids and s_w is a recoverable shear, is effectively a ratio of the principal normal stress difference at capillary wall N_{1w} and $(\sigma_{12})_w$. The separation of $2n$ and s_w concerned various investigators, notably Bagley.[11] For polymer melts, s_w should be dominant so that

$$\Delta p_{\text{ent}} \sim N_{1w} \tag{3b}$$

Unfortunately, these authors measured the Δp_{ends} of eq. (2) and not Δp_{ent} in their experiments.

Since this period various investigators have attempted to rationally determine Δp_{ent} and Δp_{exit} from basic studies. Tomita,[12] LaNieve and Bogue,[13] and Black and Denn,[14] among others, have attempted to theoretically determine Δp_{ent} by analyzing the converging flow into a die entrance with at best questionable results. Analyses for Δp_{exit} have been proved to be more straightforward because they are based on fully developed tube flow. The analyses beginning with Sakiadis,[6] Mori and Funatsu,[7] Han, Charles, and Philippoff,[9] and reaching their culmination in Davies, Hutton, and Walters[15] and Han[16] lead to the relationship

$$\Delta p_{\text{exit}} = N_{1w} + N_{2w} \tag{4}$$

$$N_{2w} = -\frac{d \, \Delta p_{\text{exit}}}{d \log(\sigma_{12})_w} \tag{5}$$

where N_{2w}, the second normal stress difference, is found to be negative. (Various investigators of N_{2w} for polymer melts have found it to be negative and between 0 and -0.5 of the value of N_{1w}.[10–18]) Han and his colleagues[9,16,19] have made extensive measurements of Δp_{exit} in polymer melts.

We have in our laboratories long been concerned with the values of Δp_{ends} and Δp_{ent} and their

TABLE I
Summary of Low-Density Polyethylene Melts Investigated

Designation	Source	Melt flow index
LDPE-A	Ube 180W	0.25
LDPE-D	Tenn. Eastman Tenite 800	1.7
LDPE-E	—	—
LDPE-F	Dow Chemical 540E	2.1
LDPE-H	Dow Chemical 610M	5.1

relationship to entrance flow characteristics and rheological properties of melts. Early studies by Bogue and his students[13,20] involved polymer solutions, and more investigations from our group involve polymer melts.[17,21-25] These studies have often considered the relationship between the vortices occurring in die entrance flow patterns and ends pressure losses. In this note, we will evaluate shear stresses σ_{12}, principal normal stress differences N_1, and Δp_{ends} on a series of low-density polyethylene melts and compare Δp_{ends} with $N_1(\dot{\gamma}_w) \equiv N_{1w}$.

EXPERIMENTAL

Two sources of experimental data were used. A series of five low-density polyethylenes, summarized in Table I, were used in this study. Non-Newtonian shear viscosities η were measured in an Instron capillary rheometer and in a Weissenberg rheogoniometer. Principal normal stress differences N_1 were measured in a Weissenberg rheogoniometer. The apparatus and techniques used are the same as those described in other papers from our group.[17,21,22] All measurements were carried out at 160°C. Rheogoniometer measurements are generally limited to low shear rate regions.

Ends pressure losses were determined using the Instron capillary rheometer by means of eq. (1b). Three 180°-entrance-angle capillary dies of diameter 0.165 cm and L/D ratios of 3.9, 7.75, and 11.7 were used. These L/D values may perhaps be criticized for being too short.

A second set of experimental data on six LDPEs were supplied by Professor C. D. Han. These were obtained on his rheometer with flush-mounted transducers.[16,19] Normal stresses were determined in his slit/capillary rheometer.

RESULTS AND INTERPRETATION

The viscosity and normal stress results for our LDPE melts are contained in a recent paper.[25] These data are generally similar to those appearing in the literature for other polyethylene melts.[16,19,21,22,27]

In Figure 1, we plot $\Delta p_{ends}/(\sigma_{12})_w$ as a function of shear rate. The data generally indicate an increasing function of similar magnitude to that reported by earlier investigations.[21-24]
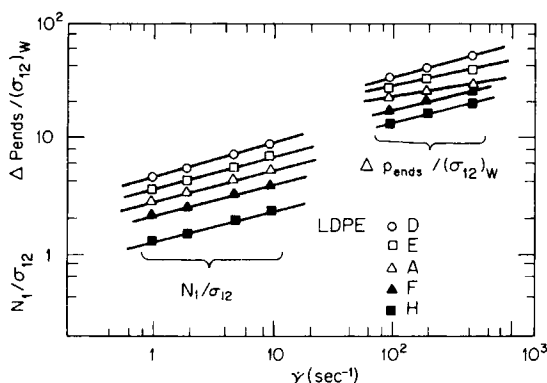

Figure 1. Reduced ends pressure loss $\Delta p_{ends}/(\epsilon_{12})_w$ and reduced principal normal stress difference N_1/σ_{12} as functions of capillary wall shear rate.

TABLE II
Ends Pressure Loss and Normal Stress Data Supplied by Han[a]

$\dot{\gamma}_w$, sec^{-1}	Δp_{exit}, psig	Δp_{ent}, psi	$\dfrac{\Delta p_{ent}}{\Delta p_{exit}}$	$(N_1)_w \times 10^{-6}$, dynes/cm^2	$\dfrac{\Delta p_{ends}}{(N_1)_w}$
a. LDPE (CX1005) at 180°C					
111.4	17.5	42.1	2.4	2.03	2.0
139.2	19.7	59.7	3.0	2.41	2.3
169.0	20.5	78.5	3.8	2.64	2.6
202.6	22.1	98.5	4.5	2.97	2.8
239.9	24.4	117.4	4.8	3.43	2.9
b. LDPE (CX1016) at 180°C					
181.6	19.3	75.9	4.0	3.45	1.9
222.7	21.8	94.2	4.3	4.02	2.0
269.6	25.5	133.9	5.3	4.77	2.3
c. LDPE (CX3020) at 180°C					
115.5	11.6	29.7	2.6	1.59	1.8
145.3	13.3	46.6	3.5	1.96	2.1
175.1	14.1	64.1	4.5	2.25	2.4
204.0	16.3	78.6	4.8	2.74	2.5
243.2	18.1	99.8	5.5	3.26	2.5
d. LDPE (NA205) at 200°C					
67.2	20.5	33.9	1.7	2.30	1.7
101.9	25.3	54.2	2.3	2.82	1.9
137.1	27.1	81.5	3.0	2.86	2.6
e. LDPE (NA244) at 200°C					
142.1	20.1	43.8	2.2	3.37	1.3
179.4	23.5	51.7	2.5	3.96	1.4
206.8	25.8	75.3	2.9	4.35	1.6
250.9	28.0	94.8	3.4	4.70	1.8
280.3	30.9	111.0	3.6	5.11	1.9
328.3	33.9	133.7	3.9	5.67	2.0
f. LDPE (NA279) at 180°C					
70.5	6.2	23.4	4.3	1.78	1.3
144.1	12.5	72.6	5.8	2.89	2.0
184.4	18.0	85.9	4.8	3.77	1.9
219.7	19.6	103.2	6.3	3.81	2.2

[a] CX1005, CX1016, and CX3020 are Chemplex low-density polyethylenes, and NA205, NA244, and NA279 are U.S.I. Chemicals low-density polyethylenes.

We also include in Figure 1 a plot of $N_{1w}/(\sigma_{12})_w$ as a function of capillary wall shear rate. Comparing the $\Delta p_{ends}/(\sigma_{12})_w$ and $N_{1w}/(\sigma_{12})_w$, which lie in different regions of the same plot, we see that there is a one-to-one correspondence in their ordering; and if the two functions are extrapolated into the same shear rate region, they differ by about a factor of 2. It follows that

$$\Delta p_{ends} = \alpha N_{1w} \qquad \alpha \sim 2 \qquad (6)$$

We now turn to Han's data which involve a quite different instrument. A value of α from 1.3 to 2.9 is obtained, with a mean value of about 2. There is a trend for Δp_{ends} to increase with shear rate for each melt.

Comparing these results with other experiments in the literature is difficult. LaNieve and Bogue's studies[13] on polymer solutions have equivalent data. From their Figures 4–7, we also see that Δp_{ends} is greater than N_{1w} and a factor of about 2 is reasonable. However, in some of these systems, Δp_{ends} is not substantially greater than that predicted for slow flow of viscous fluids. The same trend with our eq. (3a) overpredicting N_{1w} can be seen in the plots of Philippoff and Gaskins.[10]

The general reliability of eq. (4) for Δp_{exit} has been verified by Han and his co-workers[16,26] through comparison of Δp_{exit} with N_{1w} measurements from a Weissenberg rheogoniometer. We shall now evaluate Δp_{ent} using eqs. (2) and (4). This yields

$$\Delta p_{\text{ent}} = \Delta p_{\text{ends}} - \Delta p_{\text{exit}}$$

$$\cong \alpha N_{1w} - (N_{1w} + N_{2w})$$

$$\cong (\alpha - 1)N_{1w} - N_{2w} \cong (\alpha - 1 + \epsilon)N_{1w} \tag{7}$$

where we have written $N_{2w} = -\epsilon N_{1w}$ as negative. Consider the ratio

$$\frac{\Delta p_{\text{ent}}}{\Delta p_{\text{exit}}} = \frac{(\alpha - 1)N_{1w} - N_{2w}}{N_{1w} + N_{2w}} = \frac{\alpha - 1 + \epsilon}{1 - \epsilon} \tag{8}$$

If ϵ is 0.1 to 0.5 and α is 2, then the ratio is about 1.2 to 3.0. A larger value of α would predict a greater ratio. Thus, Δp_{ent} should always be larger than Δp_{exit}. This is confirmed by the data of Table II and nicely indicated by sketches in Han's papers and monograph.

It must be emphasized that the data and empirical correlation in this paper are for low-density polyethylene. It would not necessarily be expected that it would be valid for other types of polymer melts, though similar relationships might well be valid.

The influence of the ratio of reservoir to die diameter on our results requires study. Clearly it is valid for ratios of 7:1 and greater to be found in our own and Han's experiments. For small ratios the value of $\Delta p_{\text{ends}}/(J_{12})_{\omega}$ would tend to decrease.

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